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On the borderline between Science and Philosophy: A debate on determinism in France around 1880

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ABSTRACT

In the second half of the nineteenth century, a new interest in explosive chemical reactions, sudden release of energy in living beings, physical instabilities, and bifurcations in the solutions of differential equations drew the attention of some scholars. New concepts like triggering actions and guiding principles also emerged. Mathematicians, physicists, physiologists, and philosophers were attracted by this kind of phenomena since they raised a question about the actual existence of a strict determinism in science. In 1878 the mathematical physicist Joseph Boussinesq pointed out a structural analogy among physical instabilities, some essential features of living beings, and singular solutions of differential equations. These developments revived long-lasting philosophical debates on the problematic link between deterministic physical laws and free will. We find in Boussinesq an original and almost isolated attempt to merge mathematical, physical, biological, and philosophical issues into a complex intellectual framework. In the last decades, some philosophers of science rediscovered the connection between physical instabilities and determinism, both in the context of chaos theory, and in the debates on the Norton dome. I put forward a consistent historical reconstruction of the main issues and characters involved.

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In the second half of the nineteenth century, mathematicians, physicists, physicians, and philosophers were involved in debates on the complex relationship between physical and chemical instabilities, sudden release of energy in physiological processes, the questionable existence of human free will, and the determinism of scientific laws. The French mathematician Joseph Boussinesq played an important role in that process of cross-fertilisation among different disciplines: he put forward an original research programme, where different traditions of research really converged. We find the integration among mathematical researches, where singular solutions of differential equations were involved, researches in physiology, where concepts like "*Auslösung*" and "*principe directeur*" had recently emerged, and physical sciences, were transformations of energy in general, and concepts like "*trigger-work*" and "*travail décrochant*" were at stake. Boussinesq managed to offer a sophisticated and unified framework for new questions and new concepts, but his research programme faded into the background after some scientific and philosophical debates. Only after a century, some issues re-emerged in the contexts of chaos theory and philosophy of science.

The present paper puts forward a new historical reconstruction of that intellectual landscape: it aims to cast light on the network of concepts and attitudes that crossed the fields of mathematics, physics, philosophy, and life sciences. The first section is devoted to physicists and physicians who attempted to shed light on explosive processes in inanimate matter, and energy thresholds in living beings. The second section enquires into the mathematical and physical interpretations of differential equations. The third deals with Boussinesq's original integration between mathematics, science, and philosophy. In the fourth, I report on subsequent debates that involved mathematicians and scientists. In the last section I confine myself to some remarks on later philosophical debates and recent historical reconstructions.

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1. Triggering actions in physics and life sciences

In 1842 Robert Mayer stressed the two essential features of forces [Kräfte] or causes [Ursachen].[1] Firstly, they could not be destroyed, and second, they could be transformed into each other. Every cause produced a corresponding effect [Wirkung], and the effect had to equal the cause. Two years later, in a letter he sent to the physician and psychiatrist Wilhelm Griesinger, he stressed how questionable the meaning of the words "cause, effect, and transformation" really was. In the field of mental processes, could we say that "the cerebral activity" is the "cause" of the book a scholar is writing? It would be definitely pointless to say "the cause, namely the cerebral activity, transforms itself into the effect, namely the book". In the field of physical and chemical processes, the transformation of a cause into an effect was not less problematic. If a spark triggered off an explosion, might we say that the former is the cause of the latter? In this case, we cannot find an equality between cause and effect: how could the two laws Mayer himself had put forward two years before, namely the conservation of causes and their transformation into effects, be satisfied? (Mayer, 1842, pp. 4–6 and 9; Mayer, 1844, pp. 98 and 100–102).

In 1862, in a letter to the Scottish classical scholar Lewis Campbell, James Clerk Maxwell hinted at the problem of "action and reaction between body and soul". He remarked that "when a man pulls a trigger it is the gun powder that projects the bullet", and "when a pointsman shunts a train it is the rails that bear the thrust" (Maxwell, 1862, p. 712). In brief, Maxwell simply stressed that a transformation of energy should not be confused with the activation energy that triggered off that transformation.

In 1865 the French physician Claude Bernard published a book that was intended as an introduction to "experimental medicine". He stressed the peculiarity of biological processes, and at the same time the necessity of a scientific explanation: both determinism and guiding principles were at stake. The experimental method called into play determinism, because determinism was nothing else but the possibility of reproducing experiments. Bernard swung between two opposite poles: on the one hand, he put forward a strong process of reduction of life sciences to physics and chemistry, and on the other, he stressed the specific features and "the essence of life". The more demanding task was the clarification of that specific nature or essence: life required a sort of "guiding *idea*" or principle, or "creative idea", which "manifested itself in the organisation" of living beings (Bernard, 1865, pp. 6–8, 116–20, and 159–62).

In 1872, in the first part of a lecture he delivered to the German association of scientists and physicians, the German physiologist Emil Du Boi-Reymond took a different pathway. He claimed that scientific knowledge consisted in "reducing all transformations taking place in the material world to atomic motions". Since mechanical laws could be translated into the mathematical language, they could rely on "the same apodictic certainty of mathematics". The universe was ruled by "mechanical necessity": its present state could be "directly derived from its previous state", and could be looked upon as "the cause of its state in the subsequent infinitesimal time". He mentioned Laplace's Mind [ein Geist], and represented *It* as a powerful entity that would be able to "count the number of hair in our heads". Although "the human mind will always be remotely distant from this perfect scientific knowledge", what he labelled "Laplace's Mind" represented "the highest conceivable stage of our scientific knowledge" (Du Bois-Reymond, 1872, pp. 441–4 and 446).

The following year Maxwell wrote a brief essay that was not intended to be published: it was addressed to a club of scholars who had the habit of sharing their reflexions and cogitations. Once more he was interested in the relationship between mind and body, and instabilities and "singular points" were at stake. He found that "the soul of an animal" was not structurally different from "a steersman of a vessel" whose "function" was "to regulate and direct" the energy rather than "to produce" it. Instability was the key word and the key concept, and physics offered some instances of instability. Maxwell saw an intrinsic connection between instability and free will: when "we more or less frequently" found ourselves "on a physical or moral watershed", we also found the same features of the physical instability. In the moral state that corresponded to physical instability, "an imperceptible deviation" was "sufficient to determine into which of two valleys we shall descend" (Maxwell, 1873, pp. 817–21).

In the same year the Scottish physicist Balfour Stewart published an "elementary treatise on energy and its laws". The book had great success, and it was repeatedly reprinted in the following years. In the last chapter, which was devoted to "the position of life", he discussed physical and chemical instabilities, and some structural analogies with life. Both the natural world and scientific practice offered two kinds of "machines or structures": the former were characterised by their stability and "*calculability*", and the latter by their instability and "*incalculability*". Astronomical predictions represented the best instance of calculability whereas explosions, together with their "sudden and violent transmutation of energy", represented the best instance of incalculability. Living beings represented the third level of instability and incalculability after the mechanical and the chemical, and their complexity exceeded at length the complexity of first and second level machines. A different kind of action was involved indeed, because "the power of an animal, as far as energy is concerned", was not "creative, but only directive". It is worth remarking that he did not expect "to have discovered the true nature of life itself"; he had only confined himself to pointing out a very general operating principle, which offered a useful analogy (Stewart, 1873, pp. 155–9 and 161–3).

In 1844 Mayer had been puzzled by processes involving a sudden release of *force*, and in 1876 he devoted a short paper, "Ueber Auslösung", to the subject. From the outset, the two key words and concepts were "sudden release [Auslösung]" and "triggering action [Anstoß]" or impulse, and both concepts were involved in explosive processes: the latter could be looked upon as the first stage of the former. Triggering processes played an important role in life sciences, in particular "in physiology and psychology". Even in organic chemistry, and more specifically in the phenomena of fermentation, the *Auslösung* was at stake: human life depended on a network of processes of that kind (Mayer, 1876, pp. 104–6).

2. Mathematical and physical perspectives on differential equations

Instabilities and singularities were also at stake in the mathematical field. In the context of differential equations, the existence and uniqueness of solutions, and the role played by singular solutions were still under scrutiny around 1880: no systematic, conclusive, and universally accepted theory was on the stage. Only around the turn of the century a satisfactory systematisation was achieved.[2] With regard to singular solutions, some mathematicians had already come across them in the eighteenth century: among

[1] In the firs page of his paper he had claimed that "forces are causes" (Mayer, 1842, p. 1).

[2] Twenty years ago the historian of mathematics Christian Gilain stated that "the theory of ordinary differential equations still appears to be one of the most active branches of mathematics" (Gilain, 1994, p. 451).

them Brook Taylor, Alexis Clairaut, and Leonhard Euler. Euler devoted a paper to the analysis of "some paradoxes of the integral calculus" in 1759. From 1776 onwards, Lagrange enquired into the subject matter, and he managed to "create the first systematic theory of singular solutions". From the linguistic point of view, Lagrange labelled "complete solutions" what we call general solutions, and "particular integral" what we call singular solutions. The label "particular solution" was referred to "a special case of the complete solution". Unfortunately Laplace used the expressions *particular solutions* and *particular integral* "in the converse senses from Lagrange". In 1806 Siméon Denis Poisson devoted a paper to "particular solutions" and some physical applications.[3] With regard to the theorem of existence and uniqueness, in 1824 and 1835 Cauchy gave the demonstration for a specific class of equations. In 1868 Rudolf Lipschitz refined Cauchy's results, and showed that the conditions of existence were weaker than Cauchy's. Only in 1893 Emile Picard offered "the first consistent exposition of the results of existence" (Gilain, 1994, pp. 444 and 446; Grattan-Guinness, 1990, vol. 1, pp. 155 and 227, and vol. 2, p. 759).[4]

Poisson hinted at the problem in his *Traité de Mécanique* in 1833. In the context of the differential equations of motion, he analysed the simple case of a body in motion in a viscous medium. More specifically, he imagined that the body did not experience gravity, and that "the resistance of the medium" led to a deceleration that was proportional to the squared root of the instantaneous velocity. However he looked upon the physical example as "purely hypothetical": it had given him the opportunity "to show the necessity of taking into account particular solutions in differential equations of motion". He claimed that the corresponding processes "do not really happen" when we confine ourselves to forces that "really act in nature" (Poisson, 1833, pp. 249–51).[5]

Around mid-century, mathematical and physical approaches to singular solutions became different from each other, as we can easily note when we compare the mathematical treatises, which Duhamel and Cournot published in 1847 and 1857 respectively, with the physical treatise Duhamel published in 1853.[6] In Duhamel *Cours d'Analyse* the subject matter was extensively treated in the first sections of the second volume, and in Cournot's *Traité* in the fourth and seventh chapters of part VI (second volume).[7] On the contrary, in Duhamel's physical treatise the subject matter was compressed in a short section of three pages, and the physical model he discussed was an improved version of the example Poisson had put forward twenty years before. He made use of a more general viscous force of the kind $\mathrm{d}v/\mathrm{d}t = -kv^m$, where $0 < m < 1$; the singular solution $v = 0$ had to be added to the general solution in order to obtain "the complete solution of the problem under consideration" (Duhamel, 1853, pp. 328–309.

No problem emerged on the borderline between mathematics and physics in Duhamel's analysis. It seems that the first scholar who raised the question of *determinism* in connection with singular solutions of differential equations was a mathematical physicist of the Lille Faculty of Science, Joseph Boussinesq, in a brief *Note* he published in the *Comptes Rendus* of the *Académie des sciences* in 1877. He started from his "mathematical definition of *determinism*", which required that second time-derivatives of "the atoms coordinates" were functions of coordinates themselves. Physical laws were looked upon as "nothing else but specific applications" of mathematics, where the word *mathematics* corresponded specifically to "the differential equations of motions" (Boussinesq, 1877b, pp. 362–3).

Determinism corresponded to ordinary solutions of differential equations, whereas free will corresponded to the domain of singular solutions. The two different domains, determinism and free will, did not overlap with each other: he stressed that "freedom does not affect determinism", but was complementary to it. Free will came into play when physical laws "did not manage to deduce future from the present", and failed to prescribe "a completely definite pathway for natural phenomena". He dared to imagine a new kind of science which had guiding principles as its objects, and could account for the behaviour of "a *moral* and *responsible* being". He also imagined that "the singular integrals which emerge from the equations of motion for the organ of thought" could concur to set up that new body of knowledge, which was placed "at a higher level than geometry" (Boussinesq, 1877b, pp. 363–4).

Boussinesq's *Note* had been presented by his mentor Barré de Saint-Venant, who did not fail to give support to his protégé. He sent a *Note* with a similar title, *Accord des lois de la Mécanique avec la liberté de l'homme dans son action sur la matière*. He specified that no contradiction could emerge from the co-existence between "freedom in our visible actions" and "the invariability of physical laws that rule the subsequent motions of bodies". He focused on physics, in particular on explosive processes, where a little quantity of energy triggered off the transformation of huge amounts of energy. The ratio of "the work which produces the transformation of potential into actual energy" to "the amount of energy thus transformed" might be as negligible as to become zero. In living systems, the efforts of our muscles were triggered off by "the impulse of small vibrations in the nervous system which rule our free motions". According to Saint-Venant, nothing prevented us from imagining that physical actions in living systems could take place "without any expenditure of mechanical work" (Saint-Venant, 1877, p. 419 and 421–2).

At this point he was not so distant from Boussinesq: in the realm of living being, physical actions could be driven by some kind of guiding principle, which did not correspond to any measurable physical force. According to Saint-Venant, the mathematical key of those processes could be found in Boussinesq's *Note*: singular integrals were "the analytical answer" to "the necessity of a *guiding principle*". In conclusion, the supposed incompatibility between free will and "laws of motion" was not rationally founded: deterministic laws and bifurcations were generated by the same mathematical womb. Both general and singular solutions stemmed from differential equations, and no incompatibility was at stake (Saint-Venant, 1877, p. 422–3).

3. Joseph Boussinesq on differential equations, living beings, and free will

In 1878 Boussinesq published a remarkable essay under the long and demanding title *Conciliation du veritable déterminisme*

[3] In 1772 Laplace labelled simply "*solution*" or "intégrale générale" the general solution, and "intégrale particulière" every solution that "qui se trouvera de plus comprise dans l'intégrale générale". He labelled "solution particulière, toute solution qui n'y est pas comprise" (Laplace, 1772, p. 326).

[4] A general solutions of a differential equation is a function, or better a family of functions, that depend on a set of "*essential* arbitrary constants": a particular solution is obtained by giving specific values to those constants. A singular solution does not stem from the general solution: it is a function that satisfies the differential equation because "at every one of its points its slope and the coordinates of the point are the same as those of some members of the family" of particular solutions (James/James, 1992, p. 121).

[5] Poisson labelled "solution particulière" the singular solution, and "son intégrale" the general solution.

[6] After having attended the *Ecole Polytechnique*, in 1830 Duhamel taught mathematics in the same *Ecole*. Cournot was a mathematician and an economist, and he was also interested in enquiring into the foundations of scientific methodology. Cournot and Duhamel's above-mentioned treatises are the second editions.

[7] In 1841, in the second volume of his *Traité élémentaire de la théorie des fonctions et du calcul infinitésimal,* Cournot had devoted the whole 22 pages of chapter IV of part VI to singular solutions of differential equations. The first section (Cournot, 1841, II vol., pp. 271–84) was devoted to differential equations of the first order, and the second to differential equations of higher order (*Ibidem*, pp. 284–92).

mécanique avec l'existence de la vie et de la liberté morale.[8] The essay, a book indeed, was introduced by a report the philosopher Paul Janet had read before the *Academy of Moral Sciences*, and was subsequently published in the *Comptes Rendus* of the Academy. The journal also hosted a shortened version of Boussinesq's essay, which corresponded to "the philosophical section". Janet stressed that the subject matter was "very specific and technical", and he found it useful to draw the attention of philosophers to "the main idea". The core of Boussinesq's book was both mathematical and philosophical, because he described "some instances of perfect mechanical indeterminism". Some differential equations led to "*branch points* [*points de bifurcation*]", where solutions gave rise to two different pathways (Janet, 1878, pp. 3 and 12–13).

Different versions of the essay circulated in 1878 and 1879. In 1878 both the complete version (Boussinesq, 1878a, 256 pages) and the short version (Boussinesq, 1878b, 65 pages) were published: in the latter no mathematical equation appeared. In 1879 the complete version was also published in the *Mémoires de la société des sciences, de l'agriculture et des arts de Lille* (Boussinesq, 1879a, 257 pages, a one-page "Erratum" included). At that time Boussinesq had already published remarkable researches in the field of mathematical physics, and in particular fluid dynamics. In 1868 he had mathematically analysed the water flow in river bends, and in 1877 he published a treatise on the same subject (Boussinesq, 1868; Boussinesq, 1877a). Unfortunately the book was overlooked by the scientific community because theoretically or mathematically oriented scholars were not interested in his practical results, and engineers did not manage to appreciate his practical results because of the theoretical approach.[9]

Boussinesq was interested in explaining the mathematical aspect of the compatibility or reconciliation between "true mechanical determinism", on the one hand, and "the existence of life and moral freedom", on the other. Nevertheless, in his "*Avant-propos*" he devoted more than ten pages to a historical review and meta-theoretical remarks. He claimed that "the specific material features of life" could be accounted for by specific solutions of differential equations, which were "seats of convergence and bifurcation of the integrals" for those equations. He made reference to some remarks the physiologist Claude Bernard had put forward in 1867. He had assumed two kinds of "forces" inside living beings. The first had been labelled "*operating forces* [*forces exécutives*]", and they were assumed to act in the same way as in "unanimated bodies [corps bruts]", whereas the second had been labelled "*guiding or evolutionary principles*", because they had to be "morphologically active". The latter did not have to be identified with the previous vital principles, because "organic morphology" was based on "general physical-chemical forces". According to Boussinesq's reconstruction, the conflation between physical laws and the creative power of morphogenesis led to a scientific determinism which was more sophisticated than the purely mechanical determinism: even some features of "freedom" could stem from it (Boussinesq, 1878a, pp. 25–6 and 28–9).[10]

He reminded the reader that mathematicians and engineers had put forward something similar to Bernard's guiding principle. In 1861, the mathematician Antoine Cournot had spoken of "a principle of harmonic unity, global direction, and homogeneity", whereas in 1877 the mathematician and engineer Adhémar Barré de Saint-Venant had introduced a vanishing "trigger work" [*travail décrochant*], which was not so different from the small amount of force required to pull the trigger of a gun. However he found that new concepts and new words were unnecessary, and specified that a guiding principle did not need a corresponding mechanical force, however negligible it might be. The existence of "singular solutions" of differential equations was the keystone of Boussinesq's scientific and philosophical design. Singular solutions corresponded to bifurcations in natural processes, and were consistent with the emergence of life and free will. Ordinary solutions were consistent with a deterministic approach to nature, whereas singular solution could be put in connection with unpredictable processes in the context of life and mind (Boussinesq, 1878a, pp. 31–33; Saint-Venant, 1877, pp. 421–22; Cournot, 1861, pp. 364, 370, and 374).[11]

He specified that he was not in search of equations describing "*living beings*". He confined himself to "fictitious examples", which did not deal with living beings in themselves, but corresponded to some structural features of complex systems, living beings included. The first instance he put forward was also "the simplest" and "the more abstract": the motion of "a tiny heavy body along a perfectly smooth curve", where "any friction" was excluded. Singular solutions corresponded to points where the body was instantaneously at rest in a locally horizontal but unstable position. What the body might do afterwards was unpredictable, as it happened when a body was in equilibrium on the top of a dome. According to Boussinesq, only some kind of "guiding principle" was able to make the body lean towards right or left. Mathematical laws could not decide the behaviour of the body in those points: the integration of differential equations allowed the physicist only to compute the specific values of initial conditions v_0 which led to the conditions of unstable equilibrium (Boussinesq, 1878a, pp. 63–5 and 67–9).

> En résumé, *les solutions singulières cherchées correspondent aux positions d'équilibre où le mobile arrive sans vitesse*; celles-ci peuvent être, soit des sommets de la courbe, soit des points d'inflexions où la tangente est horizontale et que le mobile atteint en venant d'en bas, …. J'appellerai *points d'arrêt* de telles positions; … en sorte que le mobile pourra, au gré du principe directeur, et sans que la lois physique soit violée, s'y arrêter pendant un temps *quelconque*, puis effectuer son départ, *arbitrairement*, … […] Les point d'arrêt seront donc le *siège* du principe directeur, la région où se trouvera localisé son pouvoir, qui n'exercera que là sur le mobile (Boussinesq, 1878a, p. 70).

The number of "stopping points" depended on the shape of the trajectory, and there the system was at the mercy of "the guiding principle". The mathematical analysis of stopping points required a finer distinction between singular solutions and "asymptotic integrals", and Boussinesq undertook that analysis with great detail. Singular solutions corresponded to a body that reached the smooth top in a finite time. On the contrary, asymptotic integrals corresponded to a body that could reach the top only after an infinite time, at least "*from the abstract point of view*" (Boussinesq, 1878a, pp. 72–6).

[8] After having spent fourteen years at the University of Lille, Boussinesq held the chair of "*Mécanique physique et expérimentale*" at the Paris faculty of Science until 1896, and then the chair of "*Physique mathématique et calcul des probabilité*".

[9] For the role played by Boussinesq in the history of hydrodynamics, see Darrigol, 2009, pp. vii–ix and 233–8, and Darrigol, 2002, pp. 136 and 150; see also Apmann, 1964, pp. 427–8 and 433–4 and Guckenheimer, 1984, p. 325.

[10] Boussinesq quoted from Bernard, 1867, p. 223. He also quoted from the second volume of the treatise which Berthelot had published in 1860, *Traité de chimie organique fondée sur la synthèse*. Berthelot acknowledged that chemistry could not account for "the level of organisation" of living beings, even though "the chemical effects of life" stemmed from "ordinary chemical forces" (Boussinesq, 1878a, p. 29; Berthelot, 1860, p. 807).

[11] It is worth specifying that Saint-Venant did not make use of the expression "*travail décrochant*", even though he made use of the verb "*décrocher*".

Boussinesq's second kind of differential equation was chosen among the best-known physical models: two bodies endowed with masses M and M_1 interacted by means of a force $F(r)$ of mutual attraction, which depended on their mutual distance r. Although the problem was "less simple from the analytical point of view", it was however "the more elementary among those dealing with the motion of a real system". Singular solutions corresponded to the value r assumed when $d^2r/dt^2 = 0$ and $dr/dt = 0$ simultaneously. Because of the two-dimensional nature of the problem, those values "*did not correspond to stopping points but uniform circular trajectories*" (Boussinesq, 1878a, pp. 92–7).

On the physical side of instabilities, singular integrals could also emerge when the acceleration d^2x/dt^2 did not depend only on the coordinate x but also on the velocity $v = dx/dt$. The physical system could experience an indefinite number of transitions from ordinary trajectories to singular ones. Boussinesq remarked that, in this specific instance, the singular solutions did not depend on the choice of the initial conditions but on the analytical form of the force, in particular on the analytical dependence on velocity (Boussinesq, 1878a, pp. 82–3 and 109–11).

He was aware that both the conceptual and mathematical links between differential equations and life sciences were quite problematic. The persistence of singular states depended on "*the external conditions of isolation*", and those conditions could be easily fulfilled in the case of "a system of two atoms". On the contrary, living systems were open systems: they exchanged energy and matter with the environment. Furthermore, he was aware that the emergence of singular states from purely physical conditions could "open the door to the belief in spontaneous generation". He was not able to put forward a definite answer: he confined himself to pointing out that the initial conditions which were consistent with the establishment of singular solutions were actually "as specific as to lead to a negligible probability to be produced by pure chance". In brief, from the mathematical point of view, two contrasting features emerged from singular solutions: the improbability of the initial conditions leading to specific instabilities, and the *stability* of those instabilities. Unfortunately neither "zoologists" were able to handle the mathematical toolbox, nor mathematicians were interested in enquiring into the mathematical features of "those peculiar material systems which we call living bodies" (Boussinesq, 1878a, pp. 113–4, 116–7, and 121–30).

4. A debate on mathematics, physics, and determinism

Boussinesq and Saint-Venant explored the problematic link between specific mathematical and physical issues, and long-lasting philosophical problems. In general the boundaries between scientific practice and philosophical commitment remained quite loose until the end of the century, and during the century they could be crossed in both directions.[12] It is known that Laplace published his *Théorie analytique des probabilités* in 1812, and two years later a less demanding *Essai philosophique sur les probabilités*, where he claimed that "the most important problems of life" dealt with "problems of probability". In other words, probability was an essential feature of human knowledge. He hinted at a hypothetical "intelligence" or mighty mind, who "should know all forces acting in nature at every time", and would be able to submit that information to mathematical analysis. A scientist could not attain that kind of cleverness, even though "the perfection" of Astronomy could be looked upon as "a weak outline [faible esquisse]" of it. Although Laplace acknowledged that mathematical physics aimed at approaching the power of the superior mind he had envisioned, he also stressed that the human mind would always have been "infinitely distant" from that "intelligence". Probability was a sort of bridge that filled the gap between our body of knowledge and our ignorance (Laplace, 1825, pp. 1–4 and 6–7).[13]

The verbs he employed to describe that hypothetical, astonishing power were conditional verbs, and the sentences had the typical conditional structure *if it were ..., then it would ...*. It seems reasonable to think that the mythology of Laplacian determinism was a late reconstruction, and the physiologist Emile Du Bois-Reymond played an important role in the emergence of that mythology.[14]

Boussinesq addressed reductionism, determinism and predictability in a more sophisticated way than Du Bois-Reymond, but his bold research programme was not appreciated by his colleagues. His long essay was immediately criticised by the renowned mathematician Joseph Bertrand, who published an aggressive and sarcastic paper in the *Journal des Savants*.[15] He poked fun at "the useless array of scholarly formulae", which could "dazzle a reader who is not an expert in mathematics". According to Bertrand, that expenditure of scholarly mathematics hid a questionable superposition between the mathematical theory of mechanical systems, and concepts like choice, freedom, and will. He discussed the same physical configuration that Boussinesq had already described, and pointed out the same effects, but he found that nothing really important could be derived from those phenomena. Neither "mechanics appeared worried", nor "the science of soul had something to gain" from speculations on the possible link between mathematics and free will. He denied any possible connection between the two fields, and sharply concluded that "the mystery of soul remains unattainable". In some configurations, equations allowed the physical system to take "two different pathways", even though physical laws should only lead to "one of them". In front of the uncertainty of mathematics, physics took the lead: the least amount of force could "make the ambiguity disappear" (Bertrand, 1878, pp. 517–20).

The fact is that the supposed uncertainty of mathematics and the deterministic nature of physics opposed the traditional view he had firmly endorsed: the perfection of mathematics against the background of the coarser natural world. In reality, in Bertrand's line of reasoning, the word *mathematics* had different meanings, because different kinds of mathematics were at stake. The mathematics of "the equations of dynamics" could not attain the "absolute strictness of Euclid's theorems". In brief, three bodies of knowledge were involved: classical mathematics, mathematical physics, and physics. Mathematical physics, together with its tool box of differential equations, appeared as a shaky domain when compared to the strictness of pure mathematics and the empirical certainty of physics. According to Bertrand, Boussinesq's worst fault was his blind trust in mathematical physics: Boussinesq expected that physical systems loyally followed the differential equations when the latter "refused to decide". It was just the identification of

[12] In reality, the settlement of definite boundaries between science and philosophy was one of the achievements of scientific practice in the late nineteenth century. Even the word *scientist* does not seem suitable for some geographical context. On the process of specialisation and professionalisation that take place at the end of the nineteenth century, see for instance Ross, 1964, p. 66, and Morus, 2005, pp. 3, 6–7, 20, and 53.

[13] Laplace's, 1814 text is slightly more synthetic, and "l'induction et l'analogie" are not mentioned (Laplace, 1814, p. 1).

[14] For Ernst Cassirer and Ian Hacking's theses on the emergence of the so called *Laplacian determinism*, see the fifth section of the present paper. For a recent appraisals, see Van Strien, 2014b, p. 25.

[15] In 1874 Bertrand had been elected *sécretaire perpétuel* of the *Académie des Sciences*. He edited the *Journal des Savants* from 1865 to his death in 1900.

physical with mathematical entities that had led Boussinesq to "class molecules among living bodies". In reality, Bertrand's line of reasoning missed the point, because Boussinesq had not relied on a material analogy between singular solutions of mathematical physics and processes taking place in living beings (Bertrand, 1878, pp. 520–1).

The following month Boussinesq sent a response to the *Journal des Savants*, but the journal refused to publish it. He then sent the text to the *Revue Philosophique de la France et de l'Étranger*, which published his paper under the title *Le déterminisme et la liberté* in 1879. He found that some misunderstandings in Bertrand's paper needed to be clarified. He specified that his simplified mathematical models could not account for life in the sense of "conscious life" or life endowed with "intelligence". He had opened a field of possibilities: his models could only outline the emergence of "a basic kind of life", probably "a vegetable life" (Boussinesq, 1879b, pp. 58–60 and 63).

In the same year Boussinesq published an unsystematic collection of essays, and one of them, "*Complément à une mémoire, publiée en 1878,* …", was intended as a further elaboration of the issues he had raised. He pointed out that "a specific guiding principle" had to be postulated in science besides matter and energy. On that ground he had based a very abstract approach to life sciences, which consisted in putting forward a structural analogy between some mathematical entities and the essential features of living systems. Boussinesq relied on a very strong meta-theoretical belief: natural phenomena required natural explanations, and natural explanations could be translated into mathematical laws. Since the analogy between mathematical models and natural phenomena was structural and not ontological, Boussinesq did not dare to put forward hypotheses on the nature of guiding principles. In any case, no "finite amount of force" in the physical sense was required in order to lead matter to choose its way "at the bifurcation points" (Boussinesq, 1879c, pp. 82–3, 87–8, 90, 94, and 98).

In December 1878, Maxwell had published a three-page paper in the journal *Nature*, which was formally a review of the book *Paradoxical Philosophy* Stewart and Peter Guthrie Tait had recently published.[16] On the second page Maxwell went back to his previous cogitations on singular points or "singular phases" where "a strictly infinitesimal force" might "determine the course of the system" towards "any one of the finite number of equally possible states". He mentioned Stewart's *The Conservation of Energy* with reference to the physical side of the problem, and Saint-Venant and Boussinesq, who had analysed "the corresponding phase of some purely mathematical problems". The difference between living and dead matter involved neither matter nor "that more refined entity" called *energy*. A third level was involved, where "the application of energy may be directed without interfering with its amount". It dealt with a directive power, and the power of mind was a suitable instance of that peculiar power (Maxwell, 1878, pp. 760).

The following year Maxwell mentioned Boussinesq and Saint-Venant once more: the former was informally labelled a scholar "of hydrodynamic reputation" whereas the latter a scholar "of elastic reputation". He credited Boussinesq with having done "the whole business by the theory of the singular solutions of the differential equations of motion" in his 1878 essay. In just a few words Maxwell managed to grasp the essence of Boussinesq's research programme: "when the bifurcation of path occurs" – he wrote – the material system "ipso facto invokes some determining principle". That principle was definitely "extra physical" although in no way "extra natural", and allowed the system "to determine which of the two paths it is to follow". Maxwell also managed to grasp the difference between Stewart and Saint Venant's *physical* approaches, and Boussinesq's *mathematical* one. The first two scholars had assumed the existence of "a certain small but finite amount" of force or energy, whereas the third had "managed to reduce this to mathematical zero". Stewart's "trigger-work" or Saint Venant's "travail décrochant" were conceptually different from Boussinesq's guiding principle. In the end, Maxwell acknowledged that, however questionable Boussinesq's research programme might be, it had the advantage of relying on mathematics and natural laws, and therefore it could retrieve extraordinary or "singular" events within the domain of a scientific theory (Maxwell, 1879, pp. 756–8).

In 1880, Du Bois-Reymond commented on the debate that had been raised by his 1872 lecture. According to a "monistic point of view", our volitions are "necessary and undisputed epiphenomena of motions and rearrangements of brain molecules [Hirnmolekeln]". The universe would be a mechanism, and "in a mechanism there is no place for free will". In the last part of his lecture, the renowned physiologist remarked that Saint-Venant had introduced "the concept of triggering action [Auflösung] or *décrochement*" whereas Boussinesq had followed a slightly different pathway: he had pointed out that "some differential equations of motion" led to singular solutions, and "ambiguous or completely undetermined" states of motion followed. The main aim of the French mathematician was rightly grasped: the possibility of a better comprehension of "the multiplicity and uncertainty of organic processes". At the same time, "the German school of physiologists", he himself included, relied on "a particular kind of mechanism", and could not be satisfied with such an approach (Du Bois-Reymond, 1880, pp. 65, 74–6, 79–80, 82, and 88–9).

It is worth stressing that Boussinesq's research programme was embedded in the tradition of French mathematical physics, even though his peculiar design of integration among mathematical, scientific, and philosophical issues was not successful. In the 1870s "a further expansion" had occurred in French mathematics as a consequence of the perceived "decline of the country as a scientific centre" after the defeat in the Franco-Prussian war.[17] Mechanical instabilities and differential equations were at stake in subsequent mathematical researches. In the 1880s Henri Poincaré put forward "the geometric or qualitative theory of differential equations" in connection with "such questions as the stability of n bodes in celestial mechanics". In 1885 he studied "the equilibrium shapes" of a "homogeneous fluid mass", and "the conditions of stability" of that equilibrium: concepts like "equilibrium bifurcation" and "*pattern of bifurcation*" were at stake. Afterwards Jacques Hadamard enquired into several instances of "instability of equilibrium", where the concepts of "*attractive domain*" and "totally unstable trajectory" were involved (Grattan-Guinness, 1994a, pp. 1437–9; Gilain, 1994, p. 449; Poincaré, 1885, pp. 259, 261, and 270; Hadamard, 1901, pp. 11–12 and 14).

5. Later philosophical debates and recent rediscoveries

Structural analogies between physical and biological processes, the questionable link between mathematical physics and free will, and determinism in general attracted the attention of philosophers after having raised some debate in the scientific context. Philosophers made use of words and concepts very different from the words and concepts belonging to the scientific tradition. In general it does not seem that those scientific debates managed to open new perspectives in philosophy, even though sometimes mathematical

[16] The book was intended as a sequel to their successful *The Unseen Universe*, which had been published some years before.

[17] The Société Mathématique de France was founded in 1875.

and scientific issues became instrumental in criticising or upholding traditional philosophical thesis. Two main attitudes towards science can be singled out. On the one hand we find philosophers who were acquainted with the recent development in science, and acknowledged the philosophical meaningfulness of specific contents and theories. On the other hand, we find philosophers who underrated the scope of scientific enterprise, and expected that no scientific achievement or research could be philosophically meaningful.

Between 1878 and 1883 the debate involved Alfred Fouillée, a French philosopher with interests in politics and sociology, Charles Renouvier, a philosopher who had founded the French journal *La critique philosophique* in 1872, the French-speaking Swiss philosopher and theologian Ernest Naville, the Belgian scholar Joseph Delbeouf, and the French *polytechnicien* and historian of mathematics Paul Tannery.[18] Nevertheless the specific issues raised by the debate between Boussinesq and Bertrand slowly faded into the background. The processes of professionalisation and specialisation made the pursuit of cross-fertilisations between different bodies of knowledge ever more difficult, and discouraged further attempts in this direction.

In 1922 a new edition of Boussinesq's essay appeared, and the following year a review in the journal *Isis* also appeared. Philosophers continued to debate on determinism, and Laplace and his powerful metaphor played an important role in it, but it seems that they had forgotten Boussinesq's research programme. Nevertheless a short passage can be found in a lecture the mathematician (with wide cultural interests) Karl Pearson gave on the early history of statistics presumably in the first term of the academic session 1925–6. He remarked that Saint-Venant and Boussinesq "viewed Singular Solutions as the great solutions of the problem of Freewill" (Guinet, 1923, pp. 483–4; Pearson, 1978, pp. viii, xiv, and 360).[19]

In 1937, Ernst Cassirer claimed that Emil Du Bois-Reymond had been the first scholar to emphasise Laplace's metaphor: he had put forward a sort of "ideal" or mythology of omniscience [Allwissenheit]. After the Second world war, Karl Popper remarked that, differently from quantum physics, classical physics was usually "taken to be deterministic", but that "opposition" was "misleading". What was labelled as "Laplace's determinism" was nothing else but "a misinterpretation". In 1953 the chemist and historian of science Alwin Mittasch discussed Mayer's *Auslösung* in the first edition of a 1914 manuscript the physical chemist Wilhelm Ostwald had devoted to Mayer's, 1876 essay. Mittasch had already published a book on Mayer in 1940, where he mentioned Saint Venant's "travail décrochant" and Boussinesq's "principe directeur".[20]

In the 1970s the historian of science Mary Jo Nye reminded the readers of the existence of Boussinesq, and framed his research programme into a conceptual stream that crossed French-speaking countries between the 1870s and the 1890s. In 1983 Ian Hacking endorsed only in part Cassirer's thesis on Du Bois-Reymond's role in the emergence of a modern mythology of determinism. He traced back the origin of determinism to the seventeenth and eighteenth century. He found in Saint-Venant, his former pupil Boussinesq, and Maxwell a common attitude towards determinism, and "a completely new idea", which he qualified as "completely crazy". In 1988, the mathematician Michael Deakin explored Maxwell, Saint-Venant and Boussinesq's approaches to the "contradiction between the laws of physics and the freedom of the human will". In accordance with his *whig* perspective, he claimed that Maxwell was "distinctly modern", while Saint-Venant and Boussinesq were definitely not. In 1991 the mathematician and historian Giorgio Israel remarked that the first explicit statement of determinism "does not belong to the history of mathematical physics … but to the history of medical science". It had been the French physiologist Claude Bernard "to first introduce this term in the scientific language, at least the French one". In Boussinesq's, 1878a essay, Israel found "the widest analysis of the relationship between determinism and the theory of ordinary differential equations", but a "technically weak" solution.[21]

From the 1980s onwards, a new trend in philosophy of science led to overlooking the tradition of the discipline, and history in general. A new generation of philosophers focused on the logical foundations of science, and lost interest in the branched and tangled developments in science and philosophy. In 1986, John Earman claimed that "determinism-free will controversy" had "all of the earmarks of a dead problem" (Earman, 1986, pp. 1–2, 24, and 235).

Nevertheless remarkable philosophical work was done on determinism and the relationship between determinism and free will after Boussinesq, even though Boussinesq's specific conciliatory project was readily forgotten. More specifically, the literature on chaos theory should be mentioned: it is worth stressing that concepts like bifurcation and instability belong to the language of recent chaos theory. In the 1990s the philosophers Jesse Hobbs, James W. Garson, and John Dupré explored the possibility that free will could be consistent with the existence of chaotic rather than indeterministic systems. Freedom involved "rational control", and indeterminism could not be looked upon as a good foundation for rational control. In chaotic processes, the sensitivity to initial conditions led to both "unpredictability" and the creation of "spontaneous" structures. In the end, chaos could offer a sound framework for human behaviours, which could be qualified as "unpredictable and undetermined, while at the same time organised and intelligent". (Hobbs, 1991, p. 142, 160, 162, and 164; Garson, 1995, pp. 366–9 and 371–2; Dupré, 1996, pp. 385–7, 392–4, and 398–400).

In 2003 John Norton focused on causation and determinism in science, and discussed a simple but interesting case: a point-like mass at rest upon the top of a rotationally symmetric surface (Norton, 2003, pp. 1–5, 8–9, 13–14, and 19–21). The mass upon the dome was looked upon as a recent issue, a problem without any history. In the subsequent *philosophical* literature, it was addressed as "Norton's dome". Boussinesq and the debate that involved philosophers, scientists, and mathematicians in the second half of the nineteenth century were definitely out of reach. In 2008, the journal *Philosophy of Science* hosted a debate on Norton's dome (Norton, 2008, pp. 789–90 and 792–3; Malament, 2008, pp. 799, 808 and 815; Korolev, 2008, pp. 945 and 949–51). In the end, 130 years after Boussinesq's essay, Earman acknowledged that

[18] See for instance Renouvier, 1878, pp. 168–70; Naville, 1879, pp. 273–5, 276–7, and 281–6; Delboeuf, 1882, pp. 467, 469–70, 475, and 477–8; Delboeuf, 1882, pp. 613–16 and 623; Fouillée, 1882, pp. 585, 600–2, 604 and 608–9; Renouvier, 1883, pp. 371–2 and 375. In 1872 Fouillée had become *maitre de conférences* at the *École Normale Supérieure* in Paris. In 1878 Delboeuf was elected member of the Belgian Royal Academy of Sciences. The official biography of the Académie Royale des Sciences, des Lettres et des Beaux-Arts de Belgique qualifies Delboeuf as "philosopher, psychologist, philologist, naturalist and mathematician" (Biographie Nationale publiée par l'Académie Royale de Belgique, p. 164).

[19] As far as I know, Ian Hacking was the first to point out Pearson's remark (Hacking, 1983, pp. 464–5).

[20] Cassirer, 1937, pp. 7–9, 11, 16, and 32; Popper, 1950a, pp. 117, 122, and 128; Popper, 1950b, pp. 173 and 193; Mittasch, 1940, pp. 55 and 126; Mittasch, 1953, p. 7.

[21] Nye, 1976, pp. 274, 276, and 289–90; Nye, 1979, pp. 107, 110, and 117–9; Hacking, 1983, pp. 455–60 and 462–5; Deakin, 1988, pp. 183 and 186–9; Israel, 1991, pp. 305–7, 313, 331–3, 339, 344–5, and 352. Nye pointed out the existence of a generation of talented scientists and mathematicians who were sensitive to philosophical issues and historical developments. Israel acknowledged that the French philosopher and historian of science Georges Canguilhem had first stressed the link between "the doctrine" of determinism and Claude Bernard's scientific work (Canguilhem, 1998, pp. 64–5).

determinism in the context of physical sciences still managed "to spring surprise on us" (Earman, 2008, pp. 817–9 and 828).

Recently Marij van Strien discussed Boussinesq's research programme in connection with the 2008 debate. She found in Maxwell, Stewart, Cournot, and Boussinesq a common "concern about the irreducibility of life and the mind to physics". She also found that for Poisson, Duhamel, Boussinesq and Bertrand, "determinism was not an idea based on the properties of the equations of physics" (Van Strien, 2014c, pp. 1 and 3–4; Van Strien, 2014a, pp. 167–8, 170, 179, and 184).[22] The fact is that Poisson, Duhamel, Boussinesq, and Bertrand did not share the same view on determinism. With regard to Poisson and Duhamel, it is worth stressing that in no way were they interested in *determinism*. With regard to Boussinesq and Bertrand's views on the relationship between mathematics and the natural world, I find a subtle difference.[23] While Bertrand can really be associated with the concept of "idealisation", Boussinesq might more conveniently be associated with the concept of structural analogy. He trusted in the possibility of a structural correspondence between the essential features of singular solutions of differential equations, on the one hand, and some essential features of life and moral processes, on the other.[24] He was not interested in exploring the possible indeterminism of classical mechanics because he looked upon classical mechanics as the best implementation of determinism. Exploring indeterminism corresponded to exploring a wider domain of processes whose extension and scope was complementary to the extension and scope of mechanics.

5.1. Concluding remarks

From Mayer onwards, new concepts such as *Auslösung*, singular points, bifurcations, and *principe directeur* entered the field of scientific practice. We cannot say that new theories were put forward, but rather new attitudes, new interests, and new questions emerged. Boussinesq was the only scientist to outline a systematic approach, where a powerful analogy encompassed specific differential equations, physical instabilities, and some essential features of biological processes, the human mind included. Physical stability had its mathematical counterpart in ordinary solutions of differential equations. Physical instabilities, life and free will corresponded to singular solution. In the case of life and free will, the correspondence was formal or structural: he never spoke of or hinted at something like the equations of living processes or free actions. The structural analogy was not based on specific material similarities but on wide-scope mathematical structures. Some scholars like Maxwell found this perspective sound and attractive; others like Bertrand found it pointless and dangerous. However questionable and provisional Boussinesq's research programme might be, it realised a convergence among different traditions of research: a set of different problems that emerged from mathematics, physics, physiology, and philosophy found an original synthesis.

That specific and original integration was an isolated attempt even though the issues he addressed were anything but isolated: in the specific research fields of life sciences, physics, and mathematics those issues attracted some scholars' attention. Moreover the subject matter he explored revived philosophical debates that had traditionally crossed the history of philosophy. After many decades, the debate on the Norton dome, which took place in the first decade of the present century, and the debates on chaos theory, which took place in the last decades of the previous century, led to the re-emergence of some issues that had been raised by late nineteenth-century scholars.

[22] On Poisson and Duhamel's supposed commitment to "indeterminism" see Van Strien, 2014c, p. 13.

[23] According to van Strien, "Bertrand argued that these differential equations are only an idealisation" and "Boussinesq also described the laws of mechanic as an idealisation of physical reality" (Van Strien, 2014a, pp. 181).

[24] This is different from arguing that "he defined life mathematically" or that "it was desirable to have such an exact definition of the word *life*" (Van Strien, 2014c, p. 14).

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